Indian Statistical Institute
Back Paper Examination
Topology I - MMath I

Max Marks: 100 Time: 180 minutes.

Answer all questions. You may use Theorems stated/proved in the class after correctly stating them. You may use results not discussed in the class only after proving them.

(1) Let $\mathbb{R}_d$ denote the real line with the discrete topology. Show that the dictionary order topology on the set $\mathbb{R} \times \mathbb{R}$ is the same as the product topology $\mathbb{R}_d \times \mathbb{R}$. Compare this topology with the standard topology on $\mathbb{R}^2$. [15]

(2) Let τ be the topology generated by the following collection of subsets of $\mathbb{R}$: (i) open intervals (a, b) and (ii) sets of the form $(a, b) - K$ where $K = \{1/n\}_{n \geq 1}$. Then,
 (a) Is $(\mathbb{R}, \tau)$ connected?
 (b) Is $[0, 1]$ compact as a subspace of $(\mathbb{R}, \tau)$?
 (c) Is $(\mathbb{R}, \tau)$ path connected?

[15]

(3) Show that the product topology on $\mathbb{R}^{\mathbb{N}}$ is metrizable. [10]

(4) Let A and B be disjoint compact subspaces of a Hausdorff space X. Show that there exist disjoint open subsets U and V containing A and B respectively. [10]

(5) When do you say a space is regular? Show that a space X is regular if and only if given a point x of X and a neighbourhood U of x, there exists a neighbourhood V of x such that $\bar{V} \subseteq U$. Show that an arbitrary product of regular spaces is regular. [2+8+10]

(6) State the Urysohn lemma. Show that every connected normal space X with more than one point is uncountable. [2+8]

(7) Define the terms : homotopy equivalence, contractible, deformation retract. [2+2+2]

(8) Show that a space X is contractible if and only if the constant map $f : X \longrightarrow \{*\}$ to a one point space is a homotopy equivalence. [7]

(9) Show that S^n, $n \geq 1$ is a deformation retract of $\mathbb{R}^{n+1} - \{0\}$. [7]